TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 2 2018

Section 1 Calculator Free

Applications of Calculus

STUDENT'S NAME _____

DATE: Thursday 5 April **TIME:** 30 minutes **MARKS**: 33

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

Determine the equation of the tangent to the curve $y\sin x = x$ at the point $\left(\frac{\pi}{2}, \frac{\pi}{2}\right)$.

2. (9 marks)

(a) Determine each of the following (do not simplify)

(i) $\dfrac{d}{dx}\dfrac{x^2}{e^{\sin 3x}}$ [3]

(ii) $\dfrac{d}{dx}e^{-x}(\sin 2x - \tan 2x)$ [3]

(b) Given $f(x) = \int_x^1 (3-t)^{\frac{5}{2}}\,dt$ determine $f'(-1)$. [3]

3. (12 marks)

(a) Determine each of the following

(i) $\int \left(e^{x} + e^{-x}\right)^{2} \, dx$ [3]

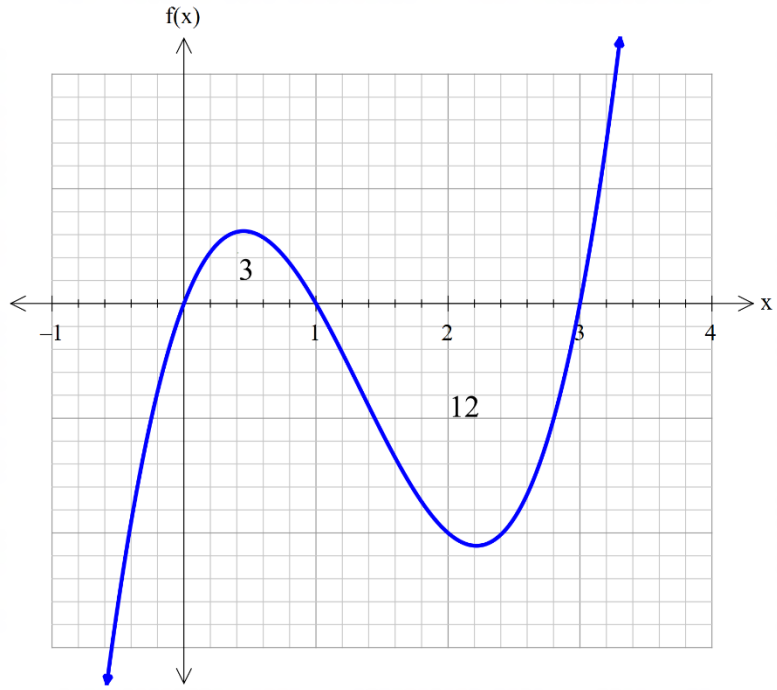
(ii) $\int 3e^{1-6x} + e \, dx$ [3]

(b) (i) determine $\frac{d}{dx} x\cos 2x$ [3]

(ii) use the result of (ii) to determine $\int 2x\sin 2x \, dx$ [3]

4. (8 marks)

The graph of y = f(x) is shown below. The size of the area of the two parts enclosed between the curve and the x-axis is shown on the graph.



Determine

(a) $\int_0^3 f(x)\,dx$ [1]

(b) $\int_0^3 |f(x)|\,dx$ [1]

(c) $\int_1^0 f(x)\,dx$ [2]

(d) $\int_1^3 (2f(x)+3)\,dx$ [4]

TRINITY COLLEGE

Mathematics Methods Units 3,4
Test 2 2018

Section 2 Calculator Assumed
Applications of Calculus

STUDENT'S NAME _____

DATE: Thursday 5 April **TIME:** 20 minutes **MARKS**: 22

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (6 marks)

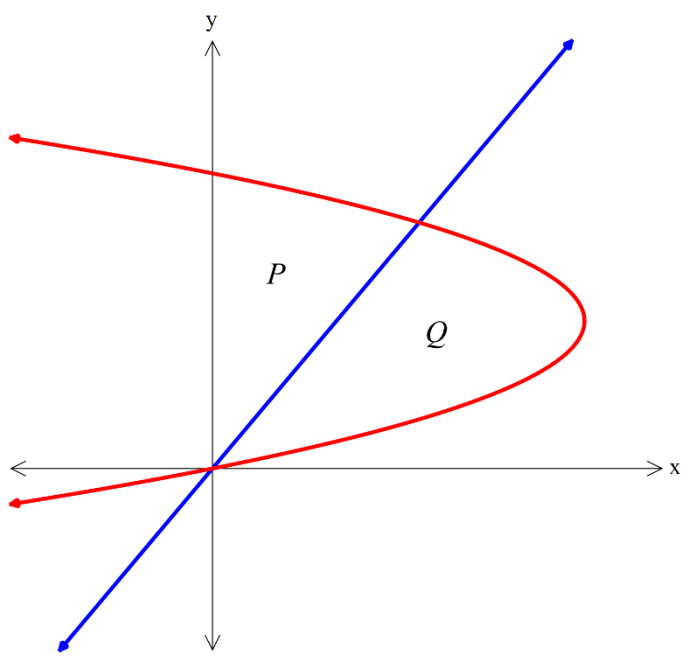
Scientists are studying a population of endangered small mammals in a protected environment. They conclude the population is increasing at a rate of given by $B'(t) = 5.2e^{0.4t}$ where t is the number of weeks since the study began.

(a) What is the change in the population in the fourth week? [3]

(b) When the study began there were 500 of these mammals. The study will conclude when the population reaches 2000. When will this occur? [3]

6. (6 marks)

In the graph shown below, Q is the area enclosed by the graphs of $y = x$ and $x = 6y - y^2$. P is the area bounded by the two graphs and the y-axis



Calculate

(a) the size of area Q [3]

(b) the size of area P [3]

7. (4 marks)

Two of the fission products of an explosion are found to decay according to the laws

$$\frac{dM_1}{dt} = -k_1 M_1 \qquad \text{where } e^{-k_1} = \frac{1}{4}$$

$$\frac{dM_2}{dt} = -k_2 M_2 \qquad \text{where } e^{-k_2} = \frac{1}{2}$$

If the initial ratio $\frac{M_1}{M_2} = 3$ what is the ratio after 6 days?

8. (6 marks)

A continuous function *f(x)* is increasing on the interval $0 < x < 3$ and decreasing on the interval $3 < x < 6$. Some of its values are given in the table below.

x	0	1	2	3	4	5	6
$f(x)$	5	16	27	32	25	0	– 49

The function *F(X)* is defined, for $0 \le x \le 6$, by $F(x) = \int_0^x f(t)dt$.

(a) At which value of *x* in the interval $0 \le x \le 6$ is *F(x)* greatest? Justify your answer. [2]

(b) At which value of *x* in the interval $0 \le x \le 6$ is $F'(x)$ greatest? Justify your answer. [2]

(c) Use the values of *f(x)* in the table to show that $48 \le F(3) \le 75$. [2]